Supporting Information of: Self-stabilizing laser sails based on optical metasurfaces

Joel Siegel\textsuperscript{1}  Anthony Y. Wang\textsuperscript{1}  Sergey G. Menabde\textsuperscript{2}  Mikhail A. Kats\textsuperscript{3}  Min Seok Jang\textsuperscript{2}  Victor Watson Brar \textsuperscript{1}  *

\textsuperscript{1}Department of Physics, University of Wisconsin-Madison, Madison WI 53606 USA
\textsuperscript{2}School of Electrical Engineering, Korea Advanced Institute of Science and Technology, Daejeon 34141, Korea
\textsuperscript{3}Department of Electrical and Computer Engineering, University of Wisconsin-Madison, Madison WI 53606 USA June 25, 2019

\*vbrar@wisc.edu
Description of Local Forces on Sail

As discussed in the main text, the reflection/refraction of light will impart optical forces on the surface. The local lateral and normal forces are shown below in Equations 1 and 2.

\[ F_x = \frac{P_i}{c} \left[ |\vec{R}| \sin(\phi - 2\theta) + |\vec{T}| \sin(\pi - \phi) \right] \]  
\[ F_z = -\frac{P_i}{c} \left[ 1 + |\vec{R}| \cos(\phi - 2\theta) + |\vec{T}| \cos(\pi - \phi) \right] \]

where \( F_x \) is the lateral, tangential force, \( F_z \) is the normal force, \( \vec{R} \) and \( \vec{T} \) are the reflected and transmitted optical rays, respectively, \( P_i \) is the incident power, \( \theta \) is the structure’s rotation relative to normal, \( \phi \) is the angle of deflection relative to surface normal, and \( c \) is the speed of light in vacuum. This analysis applies to scenarios where the reflected and transmitted phase gradients are identical, while in actual metasurfaces this assumption may not be true and will result in modification of the above equations.

In Figure S1, we show a visual representation of the motion the ICE sail can undergo (lateral offsets and rotations) and how these changes alter the lateral forces and torques on the sail. When the sail is offset, the right side of the sail is in a more intense region of the beam, so the lateral forces on the right side of the sail (restoring forces) increase in magnitude. Simultaneously, the left side of the sail is shifted to a region of lower beam intensity, creating weaker lateral forces on the left side of the sail (non-restoring). We note that while the magnitudes changes, the direction of the force vectors remain constants. The lateral motion also leads to a torque on the sail created by the left(right) side of the highly reflective center region moving to an area of higher(lower) beam intensity. When the sail is rotated (far right in S1), the angle of reflection for each point on the sail changes by \( 2\theta \) which causes a change in the magnitude and direction of the local force vectors, as detailed in equations 1 and 2. Note that the inner, highly reflective, region reflects light normal to the surface, so lateral forces in this region are only be present when the sail is rotated about the x or y axes, not when the sail is laterally shifted.

Accuracy of leapfrog integration technique

In order to verify the accuracy of our leapfrog integration technique, we compared the approximation with the exact result of the coupled oscillator system; an example is shown in Figure S2. We can see that there is a high degree of agreement between the two methods, and this agreement holds for indefinite time scales (the 5 second time scale was chosen to highlight the agreement).

‘V’-type sail motion

As described in the main text, the ‘V’-type sail reflects light at two constant angles that are equal and opposite on the left and right sides of the sail. The motion of two such sails are illustrated in Figure S3 for \( 3^\circ \) deflection angles with 95% or 65% reflection efficiency. As we can see, both sails attempt to correct for the lateral offset by moving towards the center of the beam, but they are simultaneously rotating and eventually pushed out of the beam. The difference in behaviors can be attributed to the difference in reflection coefficients: ‘V’-type Sail A is 95% reflective and ‘V’-type Sail B is 65%. The reduction in reflection coefficient leads to weaker torques, which makes the sail rotate more slowly, allowing for ‘V’-type Sail B to remain stable for slightly longer.
Figure S1: 3D representation of the ICE sail with an annular drive beam where the sail is centered in the beam (upper left), offset by $\delta$ (upper center), and rotated by $\theta$ (upper right). Lateral optical forces of the sail when it is centered in the beam (middle left), offset by $\delta$ (middle center), and rotated by $\theta$ (middle right). Normal optical forces of the sail when it is centered in the beam (lower left), offset by $\delta$ (lower center), and rotated by $\theta$ (lower right).

Figure S2: Motion (left) and rotation (right) for a single sail calculated using Leapfrog Integration technique and the exact solution derived from Mathematica.
Figure S3: Plots of the displacement (left) and rotation (right) for two different ‘V’-type sails. Each sail is four meters wide with an initial offset of 1 cm. The incident beam power is 100GW. For both sails, the incident beam is a Gaussian with a 4 meter FWHM. ‘V’-type sail A was a 95% reflective structure with a 3° deflection. ‘V’-type sail B was a 65% reflective structure with a 3° deflection.

Constants for ICE Sail

Figure S4 shows an example of the first-order dynamical force constants on an ICE sail with \( R_{\text{out}} = 0.15 \) and \( D_{\text{in}} = 2 \text{ m} \) (see Figure 3d in main text). \( C_1 \) and \( C_3 \) vary most significantly as the beam parameters shift, at least an order of magnitude shift greater than either \( C_2 \) or \( C_4 \). The magnitude of \( C_4 \) is the least dependent on the incident beam, demonstrating that the optical torques are least affected by rotations. This can be understood by considering that for small rotations, the force normal to the surface is relatively unchanged, and this force dominates the torque. In contrast, the lateral force can change more significantly as the sail is rotated, which we can see by the relative increase in magnitude of \( C_2 \).

We also note that there is a region of maximum magnitude intensity near beam separations of 2 m and FWHM of less than 1 meter for the offset dependent coefficients (\( C_{1,3} \)). This occurs because the majority of the beam power is concentrated on the border between inner and outer regions; a small offset will lead to a comparatively large change in the optical forces. As the beam width increases, this effect is lessened as there is no longer the same concentration of light.

Expanded Stability Analysis

Increasing \( R_{\text{in}} \)

In the main text, the stability analysis was restricted to a maximum \( R_{\text{out}} = 0.3 \), but in Figure S5, we show that same analysis for \( R_{\text{out}} = 0.5 \). As we can see, the increase in reflectivity leads to a decrease in stable and marginally stable regions when compared to Figure 3 in the main text. When the outer region of the sail has higher reflectivity (\( R_{\text{out}} = 50\% \)), two stable regions form.

Payload Positioning

The analysis in the main text considered the effects of beam shaping and the reflection/transmission profile of the sail, but there is one additional parameter we wish to consider: the placement of a payload, which we assume to be 5 g with a controllable mass distribution. In order to protect the payload from laser heating and prevent it from affecting the dynamical force coefficients, the payload must be placed behind the highly reflective region of the sail, but it can be arranged to optimize the moment of inertia, \( I_s \), for stability. We model two extreme scenarios, first where the payload is placed in the center of the sail which leads to a negligible change in \( I_s \), and second with the payload distributed on the outer edge of the reflective region, which increases \( I_s \) by \( \sim 29\% \); in both cases we consider sails with a 2 m inner reflective region, and an outer region with \( R_{\text{out}} = 0.3 \). The motion of these modified sails is then simulated and tested for stability as...
described in the main text, with initial offsets of 1 cm and 5 minute simulation times. The results of those simulations are shown in Figure S6, which shows that in both scenarios the margins of stability are decreased as compared to designs that do not include the payload (found in main text Fig. 3a). However, a radially distributed payload clearly shows larger regions of stability than a payload concentrated in the center. We note that while the payload has no effect on the dynamical force coefficients ($C_1, C_2, C_3, C_4$), it does effect the characteristic lateral and angular oscillation times of the sail, which modifies the overall stability. For desirable payload geometries, such changes can be compensated both by changing the reflected/refracted angle of the outer region, and by changing the reflection efficiency of the outer region.

**Noise Simulations**

To model the noise on the beam, we introduced time dependent noise; a new perturbation was included every ms of the simulations. We used the following equation to determine the noise on the system after $n$ time steps.

$$ r_n = f^n g_0 + \sqrt{1 - f^2} \sum_{i=1}^{n} g_i f^{n-i} $$

where $r_n$ is collective noise at the $n$th step, $f$ is the decay function, and $g_i$ is the noise introduced at the $i$th step. Figure S7 shows example of distortions of the beam for one time step with the undistorted beam in blue, the magnitude distortions in red, and the distorted beam in yellow. The length-scale of the noise was chosen to be 10 cm. Figure S8 shows the stability analysis over various magnitudes of noise (0.1, 0.15, 0.2, 0.3, and 5%) for a single noise profile. By the time the noise
Figure S5: Analysis of stability for 2 ICE-sail configurations driven by double-Gaussian beams with varying beam separation and FWHM for an initial offset of 1cm, 50% of the maximum allowable offset for stability. Yellow (purple) regions indicate sails configurations that are unstable (stable). The dotted red areas indicate sail configurations that satisfy the $C_1 C_4 + C_2 C_3 < 0$ condition. (a,b) are ICE sails with $R_{out} = 0.5$ and $D_{in} = 2\,\text{m}$, $1.33\,\text{m}$, respectively. The motion was simulated over a 60 second period.

Figure S6: Analysis of stability of a single ICE sail configuration with a 5g payload included at different positions behind the reflective region of the sail. The ICE sail has $R_{out} = 0.3$ and $D_{in} = 2\,\text{m}$. Yellow (purple) regions indicate configurations that are unstable (stable) when offset by 1 cm. Insets indicate the payload position on the underside of the sail.
is increased to 5%, no sails can remain stable. In the main text, 100 randomly generated noise profiles were used with an average noise value of 0.12%.

![Figure S7: Example of the noise introduced to the beam.](image)

**3D Model Comparison**

By expanding the analytic methods used in the main text, we can calculate the coefficients of a 3D, cylindrically symmetric sail. The incident beam is an annular beam, as shown in Figure S9. The cross section at the center of the sail (red line in Figure S9) is a double Gaussian, which is what we used in the 2D model in the main text. However, if we take a cross-section further to the top (black line in Figure S9), the cross section is a Gaussian beam. Our 2D model assumes that the cross section does not change, but we can see that is not accurate for a 3D model. We would also like to note that due to the cylindric symmetry, the force in the x or y directions will be dependent not only on the radial position, but also the azimuthal angle. We compare the coefficients derived from the 3D model to the 2D model in Figure S10 for an ICE sail with \( R_{\text{out}} = 0.3 \) and \( D_{\text{in}} = 2 \) m. We note that for the majority of beam combinations, the 3D model has coefficients with similar magnitudes to the 2D model, albeit with reduced magnitudes. This is a result of the azimuthal dependence of the lateral forces and the beam cross sections varying as we calculate the optical forces across the sail.

**Simulation Methods**

The phases/magnitude in Figure 4 of the main text were extracted using Lumerical FDTD where we used periodic boundary conditions along the x- and y- direction. To extract the reflected/transmitted phase, we placed a point monitor above/below the surface (where “above” means the positive z-direction). To extract the reflection/transmission, we placed a transmission monitor above/below the surface. To generate the lines shown in Figures 4b-4d (main text), we picked points to ensure that the lines had a consistent reflection (i.e. 95% or 30%) while maintaining a \( 2\pi \) phase coverage.

To arrange the resonators on the surface, we used the generalized Snell's law. We wanted a particular reflection angle at each part on the sail, and we knew the resonators had to be 1.2 \( \mu \)m apart. Thus, we knew what reflection phase we needed at each point on the sail. Then, we picked the resonator with the desired phases and placed it in the correct position. The phases of our resonators and desired are shown in Figure S11. We find that the reflected phases match with our desired behavior as they can cover \( 2\pi \). However, the transmitted phases can only cover \( \pi \), so there are portions where the phase is parallel to the ideal, but is offset. The angle of reflection/diffraction is determined by the slope of the phase gradient, so this offset to the ideal does not change the
Figure S8: Stability analysis for noise of various magnitudes ranging from 0.1% to 5%. Yellow (purple) regions indicated sail configurations that are unstable (stable). The motion was simulated over a 5 minute period.

Figure S9: Visualization of input beam magnitudes on sail.
transmitted behavior per se, however, it does introduce a phase mismatch at the boundary of these resonators, which can lead to anomalous forces/behaviors.

**Optical Force Abnormalities**

We have identified three causes for the local forces fluctuations that can, in some cases, change the direction of the lateral force from the expected behavior. These fluctuations can be caused by the presence of a metasurface edge as shown in Fig. S12(a), where light diffracts off the edge. This diffraction causes additional scattering of light from the expected behavior. In Figure S12(b), there are 'phase slips' in the metasurface, which can be seen in positions 100 and 110 µm where the lateral force changes sign in that area. The last case is caused by inter-resonator interactions as seen in Fig. S12(c). The last case is the most challenging to eliminate because it cannot be predicted by an analysis of individual resonators, or from the phase profile of the reflected/transmitted wave: although this surface produces the desired far-field profile, the lateral optical forces on individual resonators show variability, and even changes in direction.

The correlation between inter-resonator coupling and anomalous local forces can be established by analyzing the far-field scattering cross section (SCS) for each pair of neighbor resonators as demonstrated in Figure S13 (left). The net lateral force on this resonator pair can be calculated by integrating the scattered light that goes in the left and right directions. As shown in Figure S13 (right), the anomalous force fluctuation is associated with the region where scattering by neighboring pairs of resonators changes its symmetry in right-left direction. This leads to an interference between differently directed scattered waves, which causes force fluctuation. We also observed the same effect for an array with different inter-resonator distances (but same resonator sizes) in order to confirm that the effect originates from the local scattering pattern of resonators and not by the array properties (Fig. S13 (right)). These effects are compounded further by next-nearest neighbor interactions and, in general, controlling the local forces is a difficult problem to solve through
Figure S11: Phase of the reflected and transmitted light from the chosen resonators compared to the ideal phase at each resonator to produce the desired phase profile for the ICE sail simulated in the main text. The upper figure shows the phases for the entire sail, while the lower figure shows a zoomed-in portion of the sail to highlight the transmission discontinuities.
intuition.

![Image](s12.png)

Figure S12: a (Top to bottom) Cross-section of the 3D Si-resonator phased array, field profile, and optical forces at individual resonators for the edge region of the 1 degree-steering reflecting metasurface. b Same as in a, for the central region of the 2 degree-steering transmitting metasurface with phase slips and 30% reflectivity. c Same as in a, for the central region of the 1 degree-steering reflecting metasurface with continuous phase and (seemingly) near-field profiles but exhibiting the force fluctuation.

Extracting Dynamic Force Coefficients

In order to extract the dynamical force coefficient from our simulations, it is necessary to calculate the local and net optical forces as the metasurface sail is shifted and tilted within the beam, and then perform linear fits to the position/tilt vs. force/torque dependencies. In Figure S14, we show the coefficient extraction for a 3°, 95% reflective ‘V’-type sail. This sail’s motion was simulated in Figure S3, ‘V’-type sail A. Shifted simulations were performed over lateral and rotational steps of 5 μm and 0.25°. We found that the force/torque showed a linear dependence on offset/rotation angle over ranges of 40 μm and 3°. First order fits were then used to determine the effective dynamical force coefficients, C1,2,3,4. We can see that there is good agreement between our analytic models and the simulated sails. We summarize the analytic and full-wave extractions in Table S1. We note that the simulated ‘V’-type sail is a less complex metasurface than the ICE sail, so there are fewer force abnormalities to cause disagreement between our analytic model and full-wave models.

To extract the coefficients used in the main text, we performed a similar analysis to the ICE sail. Shifted simulations were performed over lateral and rotational steps of 20 μm and 1°, respectively. The larger step sizes are due to the increased computational time required for these more complex ICE sails in comparison to the ‘V’-type sails. We found that the force/torque showed a linear
Figure S13: (left) Scattered field intensity in far-field domain for a pair of resonators. (right) Lateral optical forces on individual resonators in a 1D beam-steering array (blue), and the ratio of [SCS to the right]/[SCS to the left] for each corresponding pair of neighbor resonators (red), both as a function of the resonator index in the array. Note that the ratio of light scattering to the right/left abruptly changes at the position of resonators that also show anomalous forces.

<table>
<thead>
<tr>
<th>'V' Sail</th>
<th>$C_1$ ($\frac{N}{W_m}$)</th>
<th>$C_2$ ($\frac{N}{W_{\text{deg}}}$)</th>
<th>$C_3$ ($\frac{N}{\text{deg}}$)</th>
<th>$C_4$ ($\frac{N}{W_{\text{deg}}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytic</td>
<td>-2.02E-6</td>
<td>2.25E-10</td>
<td>7.95E-9</td>
<td>1.22E-15</td>
</tr>
<tr>
<td>Full-wave</td>
<td>-1.21E-6</td>
<td>2.15E-10</td>
<td>6.76E-9</td>
<td>5.89E-15</td>
</tr>
</tbody>
</table>

Table S1: Dynamic Force Coefficients of a 'V'-type sail.

dependence on offset/rotation angle over ranges of 20 $\mu$m and up to 3°. We note that the analytic model and full-wave models are consistent up to 40 $\mu$m offsets, but that behavior is non-linear for the torque beyond 20 $\mu$m.

**Scaling Laws**

Due to computational limitations, we are unable to simulate a 4 meter wide sail, but we can derive a basic set of scaling laws that allow us to approximate the dynamic force coefficients for the 4 meter sail from a much smaller (but proportional) sail. The constant $C_1$ is derived from the lateral restoring force when the sail is offset. If the sail size is increased by a factor $S$, the sail needs to move $S$ times as far to get the same lateral response as the original sail; $C_1$ will decrease by a factor $S$. We note that the scaling of the sail will not affect the reflection/transmission when rotated, so $C_2$ should remain unchanged. However, by increasing the size of the sail, each element of the sail is now a factor of $S$ further from the center of the sail, which will influence the torques. $C_3$, the torque dependence on offset, will remain unchanged as a result. In contrast, $C_4$, the torque dependence on rotation, should increase by a factor $S$ as the force dependence on rotation is unchanged, but the distance from the center is increased by a factor of $S$. This is summarized in Table S2.

<table>
<thead>
<tr>
<th>Original</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scaled By Factor $S$</td>
<td>$C_1' = C_1/S$</td>
<td>$C_2' = C_2$</td>
<td>$C_3' = C_3$</td>
<td>$C_4' = C_4S$</td>
</tr>
</tbody>
</table>

Table S2: Summarizing the scaling laws

To test this theory, we ran simulations of an ICE sail with two different sizes: 500 $\mu$m and 1000
µm, found in Figure S16. While there is some agreement, particularly with regards to C1, we note that our predicted scaling laws can be off by as much as 40%. The most likely cause of these such effects is that in the larger sail, the force abnormalities become even more prevalent and significant, further altering the optical forces beyond our expectations.

**Temperature derived from Stefan-Boltzmann Law**

The temperature of a material absorbing radiation can be approximated with the following equation

\[ \frac{P}{A}a = \epsilon \sigma T^4 \]  

(4)

where \( P \) is the beam’s power, \( A \) is the surface area, \( a \) is the absorption, \( \epsilon \) is the emissivity, \( \sigma \) is the Stefan-Boltzmann constant, and \( T \) is the temperature of the sail. By rearranging the equation, we find that \( T = \sqrt[4]{\frac{P}{\epsilon \sigma a}} \).

**References**

1 M. Deserno. How to generate exponentially correlated gaussian random numbers.
Figure S15: Extracting the Dynamic Force Coefficients for the 5 Degree 'ICE' Sail

Figure S16: Comparison of dynamic force coefficients for 500 µm wide sail and 1000 µm wide sail